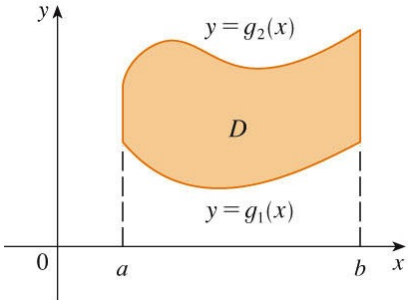
Lesson 23b. Double Integrals Over General Regions, cont.

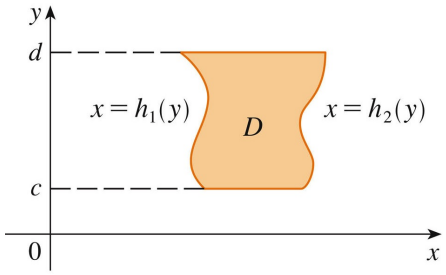
0 Warm up – last time...

- Type I region:



$$\iint_D f(x,y)\,dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\,dy\,dx$$

- Type II region:

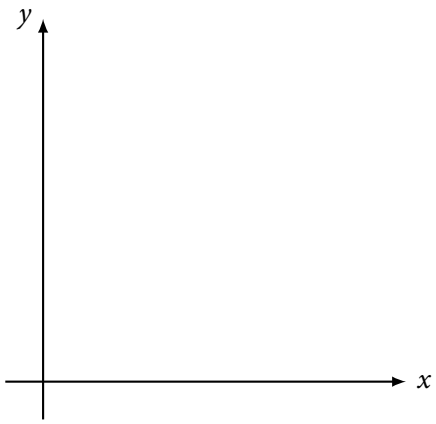


$$\iint_D f(x,y)\,dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\,dx\,dy$$

Example 1.

Sketch the region of integration D for the double integral

$$\int_0^1 \int_{x^2}^{x} \sin(y^2)\,dy\,dx$$



1 Reversing the order of integration

- Sometimes the region of integration can be a type I or a type II region

Example 2. Reverse the order of integration of the iterated integral you set up in Example 1.

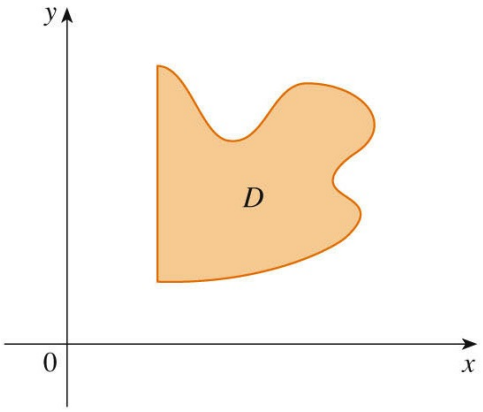
Example 3. Consider the double integral $\iint_D (x^2 + y^2)\, dA$ where D is enclosed by $x = 0$ and $x = \sqrt{1 - y^2}$. Set up this double integral as an iterated integral using both orders of integration.

Example 4. Consider the double integral $\int_0^4 \int_{\sqrt{x}}^2 f(x, y)\, dy\, dx$. Sketch the region of integration and reverse the order of integration.

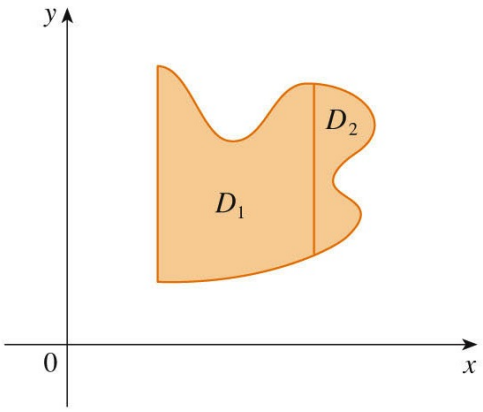
2 Combining and splitting regions of integration

- If $D = D_1 \cup D_2$, where D_1 and D_2 don't overlap except perhaps on their boundaries, then

$$\iint_D f(x, y)\, dA = \iint_{D_1} f(x, y)\, dA + \iint_{D_2} f(x, y)\, dA$$



(a) D is neither type I nor type II.



(b) $D = D_1 \cup D_2$, D_1 is type I, D_2 is type II.

Example 5. Write $\iint_D (2 - 2x - y)\, dA$ as the sum of 2 type I region iterated integrals, where D is the triangular region enclosed by $y = 0$, $y = x$, and $y = 2 - x$.

3 If we have time...

Example 6. Sketch the region of integration D for the double integral $\int_1^8 \int_0^{\ln(x)} f(x, y)\, dy\, dx$. Reverse the order of integration.